Structured Knowledge

Chapter 7
Logic Notations

Does logic represent well knowledge in structures?
Logic Notations

Frege’s *Begriffsschrift* (concept writing) - 1879:

assert P

not P

if P then Q

for every x, P(x)
Logic Notations

Frege’s *Begriffsschrift* (concept writing) - 1879:

“Every ball is red”

```
\begin{array}{c}
x \\
\hline
\text{red}(x) \\
\hline
\text{ball}(x)
\end{array}
```

“Some ball is red”

```
\begin{array}{c}
x \\
\hline
\text{red}(x) \\
\hline
\text{ball}(x)
\end{array}
```
Logic Notations

Algebraic notation - Peirce, 1883:

Universal quantifier: \( \Pi_x P_x \)

Existential quantifier: \( \Sigma_x P_x \)
Logic Notations

Algebraic notation - Peirce, 1883:

“Every ball is red”: $\Pi_x(\text{ball}_x \rightarrow \text{red}_x)$

“Some ball is red”: $\Sigma_x(\text{ball}_x \cdot \text{red}_x)$
Logic Notations

Peano’s and later notation:

“Every ball is red”: \((\forall x)(\text{ball}(x) \supset \text{red}(x))\)

“Some ball is red”: \((\exists x)(\text{ball}(x) \land \text{red}(x))\)
Logic Notations

Existential graphs - Peirce, 1897:

Existential quantifier: a link structure of bars, called line of identity, represents $\exists$

Conjunction: the juxtaposition of two graphs represents $\land$

Negation: an oval enclosure represents $\neg$
Logic Notations

“If a farmer owns a donkey, then he beats it”:
Logic Notations

\(((p \rightarrow r) \land (q \rightarrow s)) \rightarrow ((p \land q) \rightarrow (r \land s))\)
Existential Graphs

EG’s rules of inferences:

**Erasure**: in a positive context, any graph may be erased.

**Insertion**: in a negative context, any graph may be inserted.

**Iteration**: a copy of a graph may be written in the same context or any nested context.

**Deiteration**: any graph may be erased if a copy of its occurs in the same context or a containing context.

**Double negation**: two negations with nothing between them may be erased or inserted.
Existential Graphs

Prove: 

\[((p \rightarrow r) \land (q \rightarrow s)) \rightarrow ((p \land q) \rightarrow (r \land s))\]

is valid
Existential Graphs

Prove: $((p \rightarrow r) \land (q \rightarrow s)) \rightarrow ((p \land q) \rightarrow (r \land s))$ is valid
Existential Graphs

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Existential Graphs

- $\alpha$-graphs: propositional logic
- $\beta$-graphs: first-order logic
- $\gamma$-graphs: high-order and modal logic
Semantic Nets

• Since the late 1950s dozens of different versions of semantic networks have been proposed, with various terminologies and notations.

• The main ideas:

  For representing knowledge in structures
  The meaning of a concept comes from the ways it is connected to other concepts
  Labelled nodes representing concepts are connected by labelled arcs representing relations
Semantic Nets

Mammal

Person

Nose

Red

Owen

Liverpool

person(Owen) \equiv \text{instance}(Owen, \text{Person})

team(Owen, Liverpool)
Semantic Nets

- John
  - height
  - value
  - H1
  - greater-than
  - H2
  - Bill
  - height
  - value
  - 1.80
Semantic Nets

“John gives Mary a book”

give(John, Mary, book)
Frames

• A vague paradigm - to organize knowledge in high-level structures

• “A Framework for Representing Knowledge” - Minsky, 1974

• Knowledge is encoded in packets, called frames (single frames in a film)

Frame name + slots
Frames

Hybrid systems:

Frame component: to define terminologies (predicates and terms)

Predicate calculus component: to describe individual objects and rules
Conceptual Graphs

- CG = a combination of Perice’s EGs and semantic networks.
Conceptual Graphs

- 1970's: seriously working on CGs
- 1976: first paper on CGs
- 1981-1982: meeting with Norman Foo, finding Peirce’s EGs
- 1984: the book coming out
- CG homepage: http://conceptualgraphs.org/
Simple Conceptual Graphs

Concept type (class)

Individual referent

Generic referent

Concept

Relation

Relation type

CAT: tuna

On

MAT: *

1

2
Ontology

• **Ontology**: the study of "being" or *existence*

• **An ontology** = "A catalog of types of things that are assumed to exist in a domain of interest" (Sowa, 2000)

• **An ontology** = "The arrangement of kinds of things into types and categories with a well-defined structure" (Passin 2004)
Ontology

top-level categories

domain-specific
Ontology

Aristotle's categories

- Being
  - Substance
  - Accident
    - Property
      - Inherence
        - Quality
          - Activity
      - Directedness
        - Quantity
          - Passivity
      - Containment
        - Movement
          - Intermediacy
            - Spatial
              - Having
            - Temporal
              - Situated
Ontology

Geographical categories

- Area
  - Block
  - Terrain
  - Country
  - Wetland
  - Mountain

- Point
  - Dam
  - Town
  - Bridge
  - Airstrip
  - Heliport

- Line
  - On-Land
    - Road
    - Border
  - On-Water
    - River
  - Railroad
  - Power-Line
Ontology
CG Projection

PERSON: john

1

Has-Relative

2

PERSON: *

PERSON: john

1

Has-Wife

2

WOMAN: mary
Nested Conceptual Graphs

It is not true that cat Tuna is on a mat.
Nested Conceptual Graphs

Every cat is on a mat

coreference link
Nested Conceptual Graphs

Julian could not fly to Mars

Poss
PERSON: julian  Fly-To  PLANET: mars
Past

Julian could not fly to Mars
Nested Conceptual Graphs

Tom believes that Mary wants to marry a sailor
Exercises

• Reading:
