Processor Organization

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Outline

- Criteria:
  - Diameter, bisection width, etc.

- Processor Organizations:
  - Mesh, binary tree, hypertree, pyramid, butterfly, hypercube, shuffle-exchange
Criteria

- **Diameter**
  - The largest distance between two nodes
  - Lower diameter is better

- **Bisection width**
  The minimum number of edges that must be removed in order to divide the network into two halves (within one)

- **Number of edges per node**

- **Maximum edge length**
Mesh (1)

- **Q-dimensional** lattice
- Communication is allowed only between neighboring nodes. Interior nodes communicate with 2q other nodes.
Mesh (2)

- Q-dimensional mesh with $k^q$ nodes
  - Diameter: $q(k-1)$
  - Bisection width: $k^{q-1}$
  - The maximum number of edges per node: $2q$
  - The maximum edge length is a constant
Binary Tree

- **Depth k-1**: $2^k - 1$ nodes
- **Diameter**: $2(k - 1)$
- **Bisection width**: 1
- **Length of the longest edge**: increasing
Fat Tree

- Bandwidth problem on binary tree
Hypertree of degree $k$ and depth $d$: a complete k-ary tree of height $d$.
A 4-ary hypertree with depth $d$ has $4^d$ leaves and $2^d(2^{d+1}-1)$ nodes in all.

- Diameter: $2d$
- Bisection width: $2^{d+1}$
- The number of edges per node $\leq 6$
- Length of the longest edge: increasing
Pyramid

- **Size** $k^2$: base a 2D mesh network containing $k^2$ processors, the total number of processors = $(4/3)k^2 - 1/3$

- A pyramid of size $k^2$:
  - Diameter: $2\log k$
  - Bisection width: $2k$
  - Maximum of links per node: 9
  - Length of the longest edge: increasing
Butterfly (1)

- \((k+1)2^k\) nodes divided into \(k+1\) rows (rank), each contains \(n=2^k\) nodes.
- Ranks are labeled 0 through k
- \(\text{Node}(i,j)\): j-th node on the i-th rank
- \(\text{Node}(i,j)\) is connected to two nodes on rank i-1: \(\text{node}(i-1,j)\) and \(\text{node}(i-1,m)\), where m is the integer found by inverting the i-th most significant bit in the binary representation of j
- If \(\text{node}(i,j)\) is connected to \(\text{node}(i-1,m)\), then \(\text{node}(i,m)\) is connected to \(\text{node}(i-1,j)\)
- Diameter = \(2^k\)
- Bisection width = \(2^k\)
- Length of the longest edge: increasing
Butterfly (2)

Node(1,5): i=1, j=5

\[ j = 5 = \begin{array}{c} 101 \\ \end{array} \text{(binary)} \]

\[ \downarrow \]

\[ i=1 \]

\[ \begin{array}{c} 001 \\ \end{array} = 1 \]

Node(1,5) is connected to node(0,1)
Hypercube (1)

- $2^k$ nodes form a $k$-dimensional hypercube
- Nodes are labeled 0, 1, 2,..., $2^k - 1$
- Two nodes are adjacent if their labels differ in exactly one bit position
- Diameter = $k$
- Bisection width = $2^{k-1}$
- Number of edges per node is $k$
- Length of the longest edge: increasing
Hypercube (2)
Hypercube (3)

- $5 = 0101$
- $1 = 0001$
- $4 = 0100$
- $13 = 1101$
Others

- Torus
  - http://clusterdesign.org/torus/
- Cube-Connected cycles
- Shuffle-Exchange
- De Bruijn