MULTI-LEVEL LOGIC OPTIMIZATION(CONT)

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CONTENT

- Boolean division
- Don't care based optimization

BOOLEAN DIVISION

- Algebraic division:
- Example: f = abd+cd + abe+ace, assumed g = ab + c
  = d(ab + c) + abe + ace
- More optimal: f = (ab + c)(ae + d)
- Why?

- Algebraic division: f = h.g + r
- h and g are orthogonal

BOOLEAN DIVISION

- Boolean division: f = h.g + r
- **h** and **g** can share:
- A common literal **x**
  a + bc = (a + b).(a + c)
- Literal x and **x'**
  ab + a'c + bc = (a + b)(a' + c)

BOOLEAN DIVISION

- Provided f, g; reexpress f = h.g + r (minimized)

- Label g as an input G
- Construct a function F = { Don't care, G != g ; f, G = g }
- DC set of F = G ^ g' = G.g' + G'.g
- ON set of F = f . (G.g + G'.g') = f . (G.g' + G'.g)'
- Apply some known optimization algorithm on F
- Note: F === f, because G = g

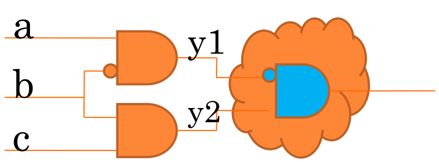
DON'T CARE BASED OPTIMIZATION

Two types of Don't Care conditions:

- External Don't Cares: defined by user, example: the DC-set
- Internal Don't Cares: exist because of the structure of the boolean network. Two types of Internal Don't Cares:
  - Satisfiablility don't care
  - Observability don't care

SATISFIABILITY DON'T CARES

If input cannot occur, don't care the output



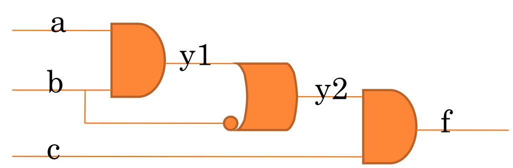
y1	y2	f
0	0	0
0	1	1
1	0	0
1	1	0

Never occur →

y1	y2	f
0	0	0
0	1	1
1	0	0
1	1	x

→ f = y2

SATISFIABILITY DON'T CARES



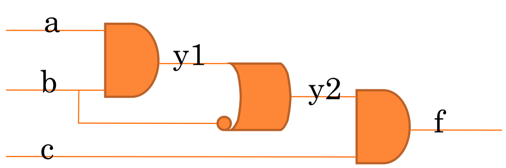
- Yj = Fj(x, y)
- Example: Y2 = OR(~b, Y1)
- Which configuration of inputs of Y2 cannot occur?

Answer: Y1.F1' + Y1'.F1

Y1.(a' + b') + Y1'.a.b

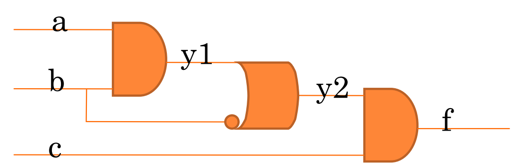
Example: Y1 = 1, b = 0 cannot occur

SATISFIABILITY DON'T CARE



- Satisfiability Don't Care Set:
- $SDC = \sum_j (Yj.Fj' + Yj'.Fj)$
- Optimize a node N:
- Use Satisfiability Don't Care Set of Nodes that fan-out to N

OBSERVABILITY DON'T CARES

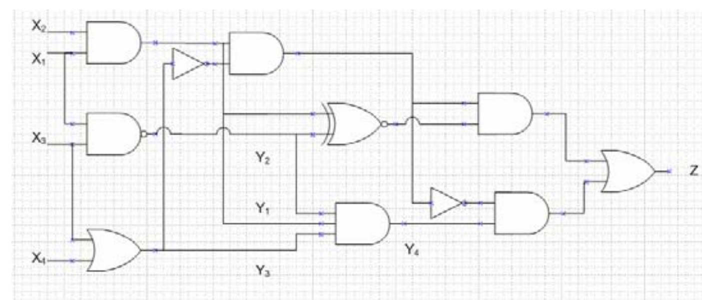


- Says: The output F can observe the input X (X can be observed at output F) if changes of X make F changed
- Example: if c = 1 then y2 can be observed at F
  if c = 0 then y2 cannot be observed at F

OBSERVABILITY DON'T CARES

- Define Observability of node Yj:
- $\partial F_k / \partial Y_j = F_{kYj}$ XOR $F_{kYj'}$ (Boolean difference)
- $\partial F_k / \partial Y_j = 0$ (or $F_{kYj} = F_{kYj'}$) : Y_j cannot be observed at F
- $F_{kYj} = F_{kYj'}$: Observability Don't Care condition for Y_j
- Observability Don't Care Set of node Y_j
- $ODC = \Pi_{\text{all outputs}}(F_{kyj} = F_{kyj'}) = \Pi_{\text{all output}}(\partial F_k / \partial Y_j)'$

DON'T-CARE GENERATION



$Y_1 = x_1.x_2$

$Y_2 = \overline{x_1.x_3}$

$Y_3 = x_3 + x_4$

$Y_3=0 \rightarrow Y_2=1$

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DON'T-CARE GENERATION

Local satisfiability don't–care set for node 4

$$SDC_4 = \overline{y_2}.\overline{y_3}$$

Sum of product representation for node 4

$$F_4 = y_1.y_2.y_3$$

DON'T-CARE GENERATION

The function at the primary output Z

$$Z = \overline{x1.x2.\overline{x3}.\overline{x4}}.\ y4 + \overline{x1 \oplus x2}.x1.x2.\overline{x3}.\overline{x4}$$

$$Z = (\overline{x1} + \overline{x2} + x3 + x4)\ y4\ + x1.x2.\overline{x3}.\overline{x4}$$

Cofactor of z with respect to y4 are

$$Z_{y4} = \overline{x1} + \overline{x2} + x3 + x4 + x1.x2.\overline{x3}.\overline{x4} = 1$$

$$Z_{\overline{y4}} = x1.x2.\overline{x3}.\overline{x4}$$

DON'T-CARE GENERATION

Finally, ODC_4 can be expressed as

$$ODC_4 = \overline{z_{y4} \oplus z_{y4}} = x1.x2.\overline{x3}.\overline{x4}$$

110 is not in the local care set

$$SDC_4\ U\ DC_4 = \overline{y_2}.\overline{y_3} + y_1.y_2.\overline{y_3}$$

$$F_4 = y_1.y_2$$

DON'T-CARE GENERATION

1. Select a node i in the Boolean network
2. For each primary output z_k, compute cofactor z_k respect to y_i and $\overline{y_i}$
3. Compute

$$C_i = \sum_{\text{All outputs } k} z_{yi} \oplus z_{\overline{yi}}$$

4. $C_i \rightarrow LC_i$
5. Minimize F_i with local care set SDC_i U $ODC_i = \overline{LC_i}$

RANGE COMPUTATION

Characteristic function

Let $f : B^N \rightarrow B^M$

Let $A \subseteq B^N$, $\eta_A : B^N \rightarrow \{0,1\}$ which is defined as

$\eta_A(x) = 1$ if $x \in A$, else $\eta_A(x) = 0$

Smoothing function

Let $f : B^N \rightarrow B$ and $x=(x_1,x_2,\ldots,x_n)$

$$S_x f = S_{x1} S_{x2} \ldots S_{xn} f$$
$$S_{xi} f = f_{xi} + f_{\overline{xi}}$$

RANGE COMPUTATION

Transition relation

Let $f : B^N \rightarrow B^M$

$\Rightarrow$ $F : B^N \times B^M \rightarrow B$

$F(x,y) = 1 \Leftrightarrow (x,y) \in B^N \times B^M$ and $y = f(x)$

$\Leftrightarrow F(x,y) = \prod_{1 \le i \le M} \overline{(y_i \oplus f_i(x))}$

$\Rightarrow$ $f(A) = \{y : \exists x (x \in A) \square F(x,y) = 1\}$

$\Rightarrow$ $f(A)(y) = S_x(F(x,y).A(x))$

RANGE COMPUTATION

Example:

$y1 = x1.x2$
$y2 = \overline{x1} + \overline{x3}$
$y3 = x3 + x4$

$\Rightarrow$ $F(x,y) = \overline{y1 \oplus (x1.x2)}.\ \overline{y2 \oplus (\overline{x1} + \overline{x3})}.\ \overline{y3 \oplus (x3+x4)}$

$C_4 = A(x) = \overline{x1} + \overline{x2} + x3 + x4$

$\Rightarrow$ $LC_4 = f(A)(y) = S_x(F(x,y).A(x))$

$\Rightarrow$ $\overline{LC_4} = \overline{y2}.\overline{y3} + y1.y2.\overline{y3}$