Solution 1

Regular expressions and context-free grammars

1. Describe the languages denoted by the following regular expressions:
   a. \( 0(0|1)^*0 \)
      The language contains binary numbers that starts with a 0 and ends with another 0.
   b. \( (0|1)^*0(0|1)(0|1) \)
      The language contains binary numbers that ends with 000,011,010, or 001.
   c. \( 0*10*10*10* \)
      The language contains binary numbers that have exactly 3 digits.

2. Let \( \Sigma = \{a,b\} \). Write regular expressions for the languages over \( \Sigma \) that contain:
   a. All strings beginning with \( ab \).
      \( ab(a|b)^* \)
   b. All strings that contain exactly two \( a \)'s.
      \( b^*ab^*ab^* \)
   c. All strings in which every \( a \) is followed by a \( b \).
      \( (b|ab)^+ \)

3. Floating-point decimals consist in an integer part, a decimal part and an exponent part. The integer part is a sequence of one or more digits. The decimal part is a decimal point followed by zero, one or more digits. The exponent part is the character \( e \) or \( E \) followed by an optional + or - sign, followed by one or more digits. The decimal part or the exponent part can be omitted, but not both. Write a regular expression to specify floating-point decimals.

   - \( \text{digit} \rightarrow 0|1|2|3|4|5|6|7|8|9 \)
   - \( \text{integer_part} \rightarrow \text{digit}^+ \)
   - \( \text{decimal_part} \rightarrow . \text{digit}^* \)
   - \( \text{exponent_part} \rightarrow \text{(E|e)(+|-)?digit}^+ \)
   - \( \text{floating-point} \rightarrow \text{integer_part decimal_part (exponent_part)?} \)
     \[ \text{floating-point} \rightarrow \text{integer_part} \text{ exponent_part} \]

4. Consider the following grammar
   \[ S \rightarrow ( L ) | a \]
   \[ L \rightarrow L , S | S \]
   a. What are the terminals, nonterminals, and start symbol?
      Terminals: ( ) a ,
      Nonterminals: S L
      Start symbol: S
   b. Find parse trees for the following sentences:
      i. (a, a)
c. Construct a leftmost derivation for each of the sentences in (b) 
\[ S \rightarrow (L) \rightarrow (L,S) \rightarrow (S,S) \rightarrow (a,S) \rightarrow (a,a) \]
d. Construct a rightmost derivation for each of the sentences in (b)

S → (L) → (L,S) → (L,a) → (S,a) → (a,a)

S → (L) → (L,S) → (L,(L,S)) → (L,(L,a)) → (S,(a,a)) → (a,(a,a))

S → (L) → (L,S) → (L,(L,S)) → (L,(L,a)) → (S,(a,a)) → (a,(a,a))

5. Construct a grammar for regular expressions.

R → R ’|’ T | T
T → T F | F
F → a | ’ε’ | ( R) | F*