Processor Organization

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Outline

- Criteria:
  - Diameter, bisection width, etc.
- Processor Organizations:
  - Mesh, binary tree, hypertree, pyramid, butterfly, hypercube, shuffle-exchange

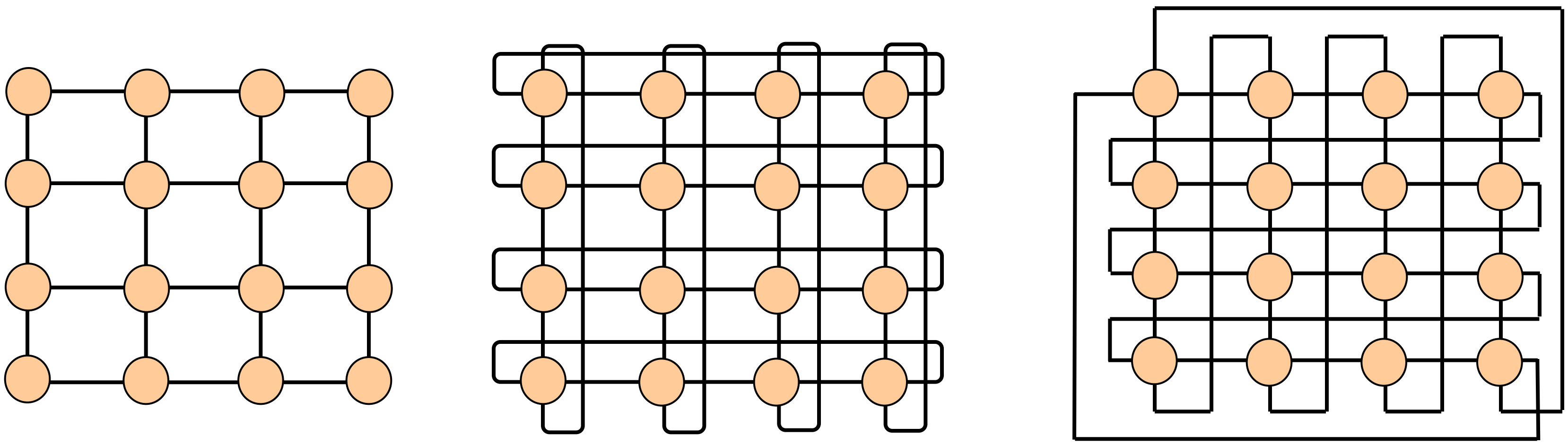
Criteria

- Diameter
  - The largest distance between two nodes
  - Lower diameter is better
- Bisection width

  The minimum number of edges that must be removed in order to divide the network into two halves (within one)
- Number of edges per node
- Maximum edge length

Mesh (1)

- Q-dimensional lattice
- Communication is allowed only between neighboring nodes. Interior nodes communicate with 2q other nodes.

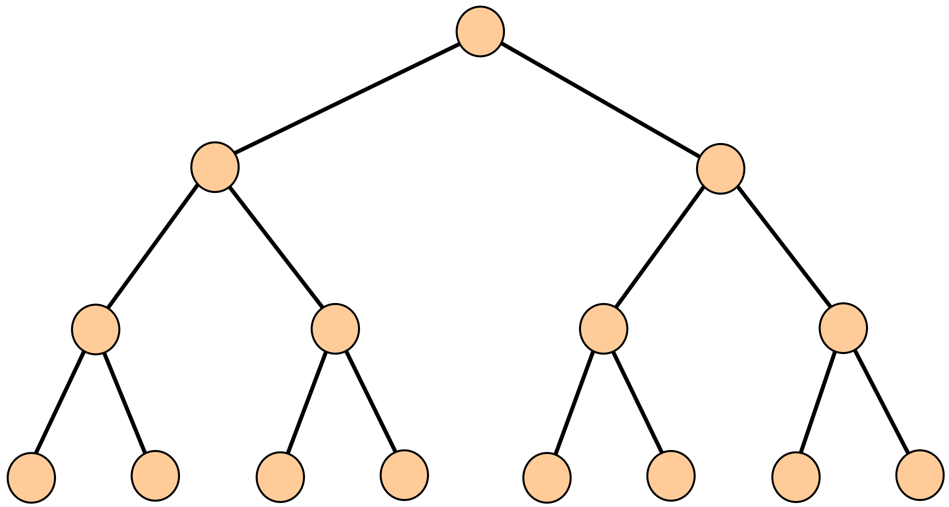


Mesh (2)

- Q-dimensional mesh with k^q nodes
  - Diameter: $q(k-1)$
  - Bisection width: k^{q-1}
  - The maximum number of edges per node: $2q$
  - The maximum edge length is a constant

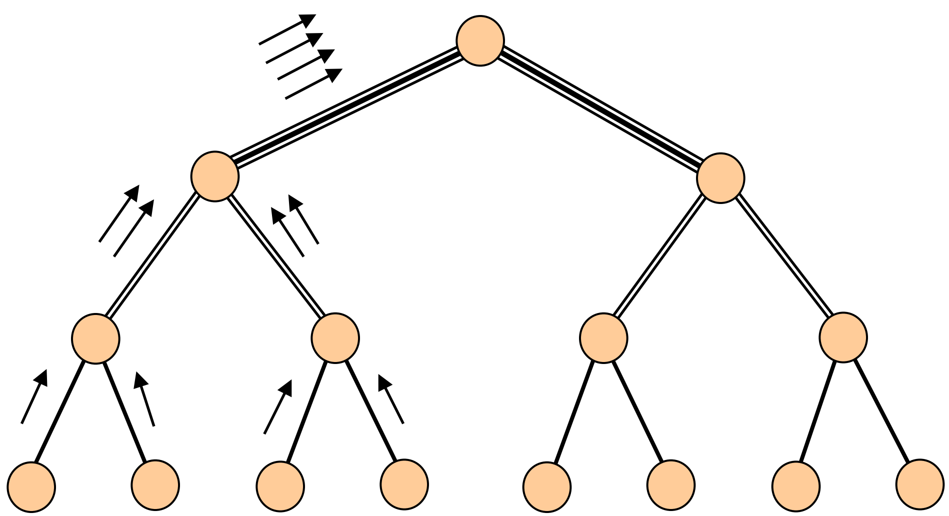
Binary Tree

- Depth k-1: 2^k-1 nodes
- Diameter: 2(k-1)
- Bisection width: 1
- Length of the longest edge: increasing



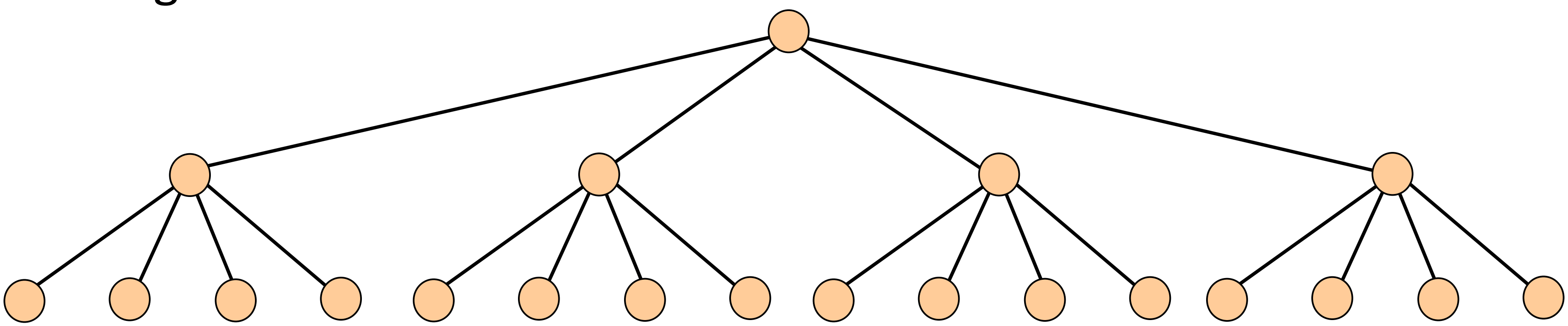
Fat Tree

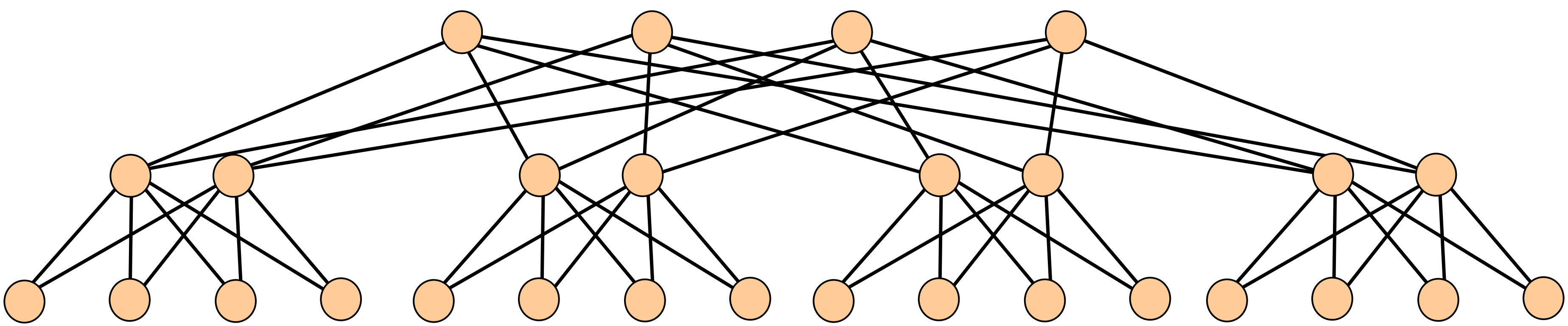
- Bandwidth problem on binary tree



Hypertree (1)

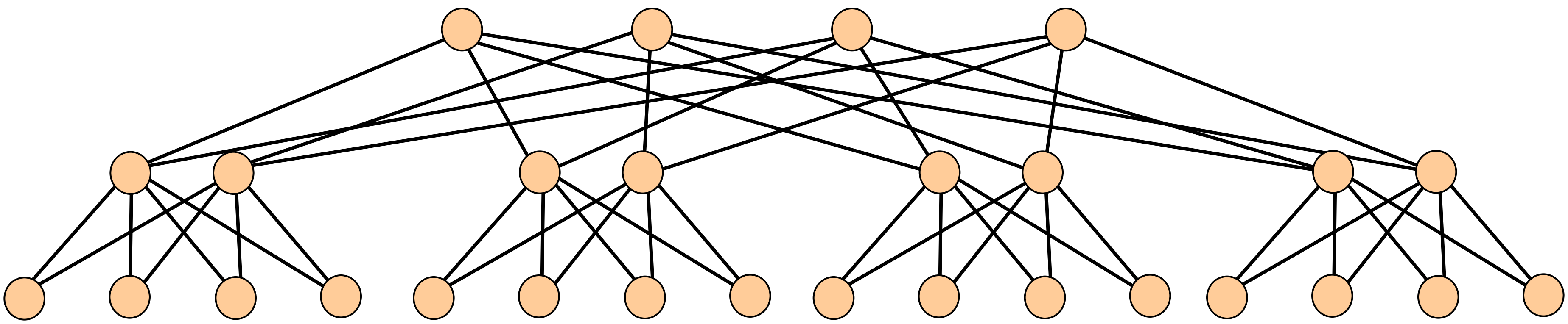
- Hypertree of degree k and depth d: a complete k-ary tree of height d.





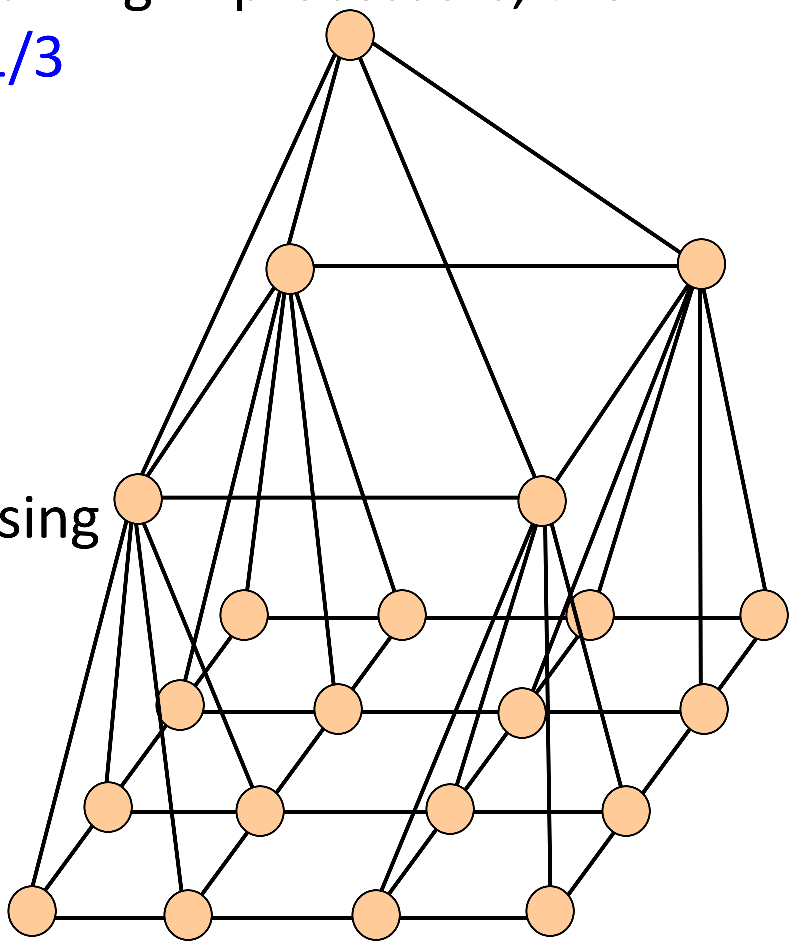
Hypertree (2)

- A 4-ary hypertree with depth d has 4^d leaves and $2^d(2^{d+1}-1)$ nodes in all
  - Diameter: 2d
  - Bisection width: 2^{d+1}
  - The number of edges per node ≤ 6
  - Length of the longest edge: increasing



Pyramid

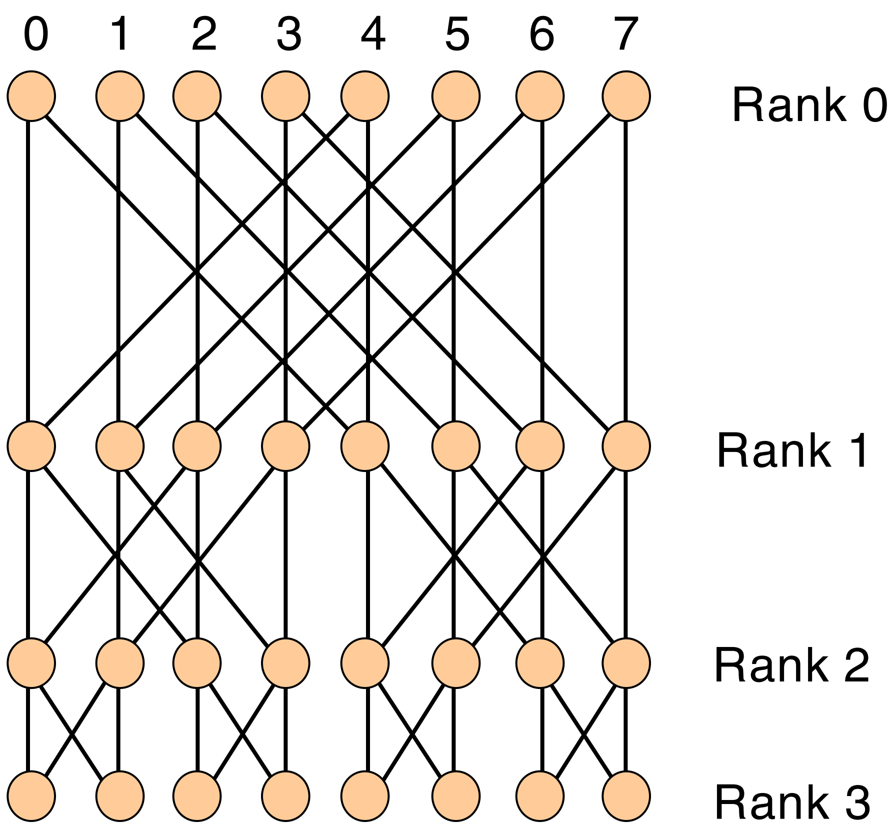
- Size k^2: base a 2D mesh network containing k^2 processors, the total number of processors=$(4/3)k^2 - 1/3$
- A pyramid of size k^2:
  - Diameter: 2logk
  - Bisection width: 2k
  - Maximum of links per node: 9
  - Length of the longest edge: increasing



Butterfly (1)

- $(k+1)2^k$ nodes divided into k+1 rows (rank), each contains $n=2^k$ nodes.
- Ranks are labeled 0 through k
- Node(i,j): j-th node on the i-th rank
- Node(i,j) is connected to two nodes on rank i-1: node(i-1,j) and node (i-1,m), where m is the integer found by inverting the i-th most significant bit in the binary representation of j
- If node(i,j) is connected to node(i-1,m), then node(i,m) is connected to node(i-1,j)
- Diameter=2k
- Bisection width=2^k
- Length of the longest edge: increasing

Butterfly (2)



Node(1,5): i=1, j=5

j = 5 = **101** (binary)

↓ i=1

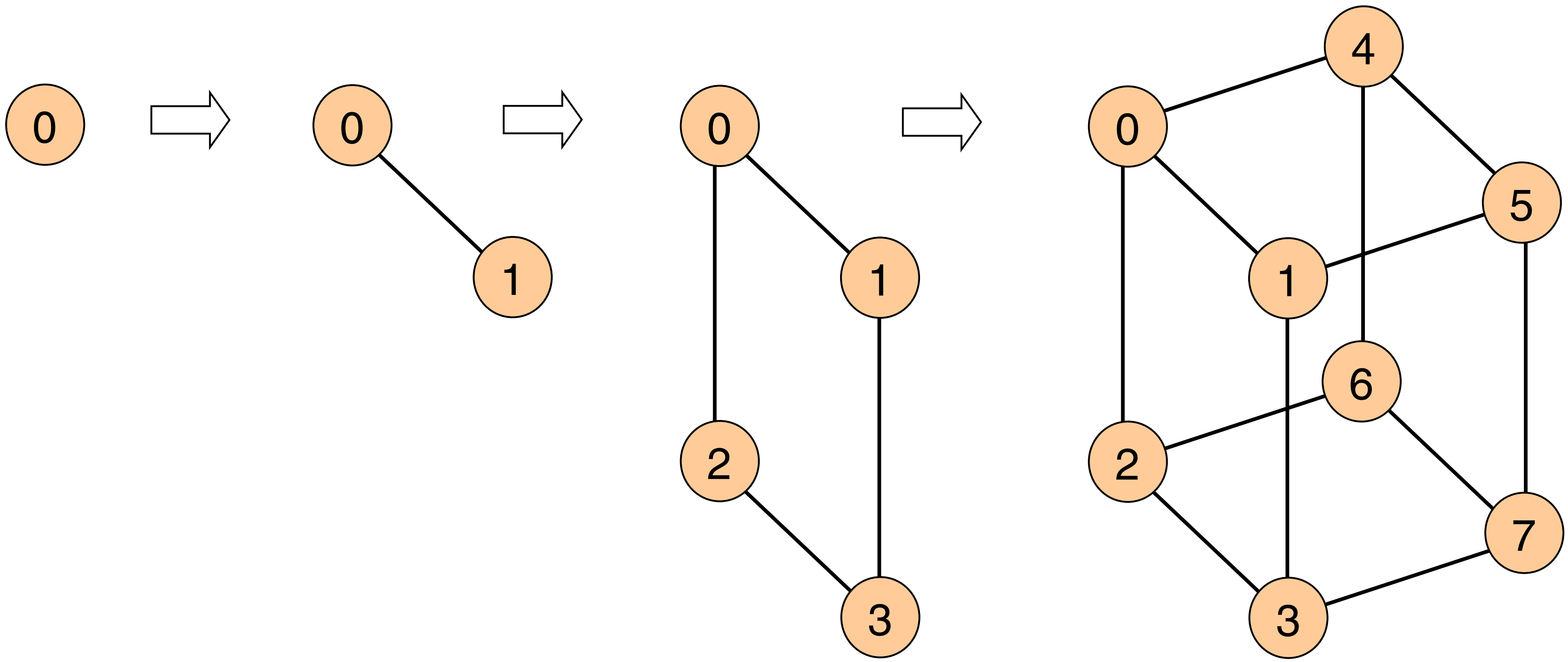
001 = 1

Node(1,5) is connected to node(0,1)

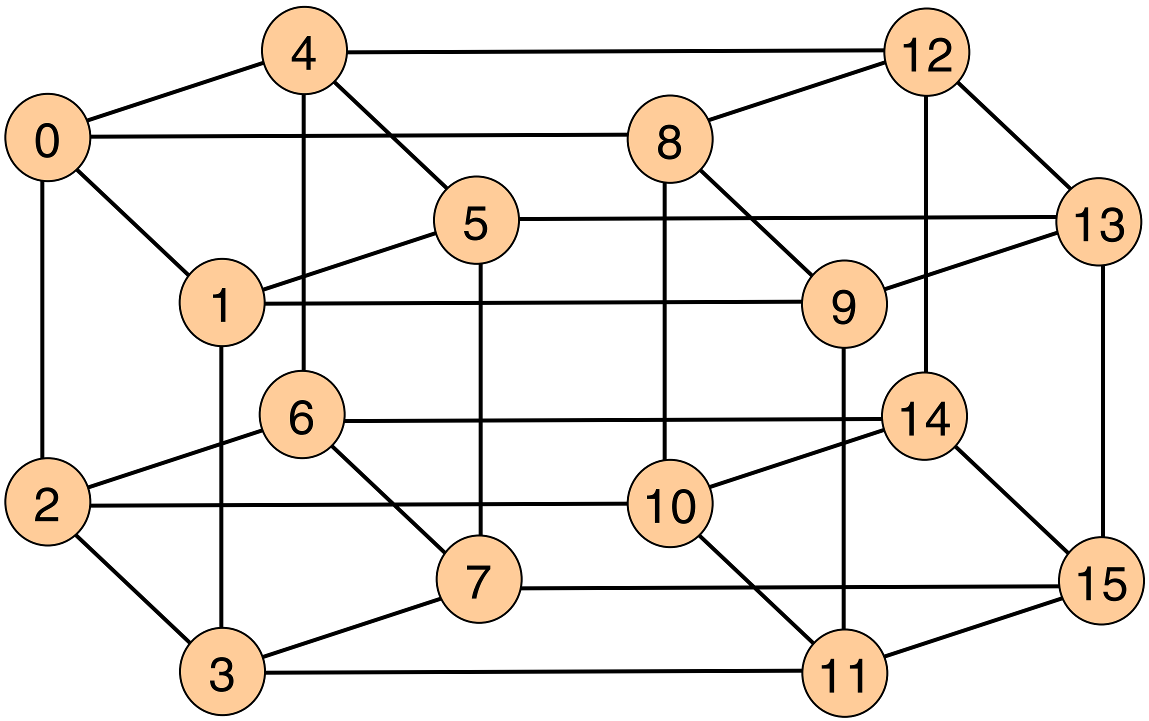
Hypercube (1)

- 2^k nodes form a k-dimensional hypercube
- Nodes are labeled 0, 1, 2,…, 2^k-1
- Two nodes are adjacent if their labels differ in exactly one bit position
- Diameter=k
- Bisection width= 2^{k-1}
- Number of edges per node is k
- Length of the longest edge: increasing

Hypercube (2)



Hypercube (3)



- 5 = **0101**
- 1 = **0001**
- 4 = **0100**
- 13 = **1101**

Others

- Torus

  http://clusterdesign.org/torus/

  http://www.fujitsu.com/global/about/tech/k/whatis/network/

- Cube-Connected cycles
- Shuffle-Exchange
- De Bruijn