Airline Fleet Assignment Problem

Tran, Van Hoai

Faculty of Computer Science & Engineering
HCMC University of Technology

E-mail: hoai@cse.hcmut.edu.vn
Homepage: http://www.cse.hcmut.edu.vn/~hoai

Reference
Handbook of Optimization - Optimization Applications in the Airline Industry

2012-2013
**Problem definition**

**Definition**

Assign aircraft types to flight legs such that *contribution* is maximized.

Factors influence fleet assignment:
- Passenger demand
- Seat capacity
- Operational costs
- Availability of maintenance

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**Diagram:**

- **Schedule Design**
  - **Fleet Assignment**
    - **Aircraft Routing**
      - **Crew Scheduling**

**Notes:**
- Select optimal set of *flight legs* in a schedule
- A flight specifies origin, destination, and departure time
- Contribution = Revenue - Costs
- Assign crew (pilots and/or flight attendants) to flight legs

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Aircraft must circulate in the network of flights
⇒ A network built on flight schedule for every fleet type
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- Time span is 24 hours or any schedule horizon
- In this lecture, we consider repeating daily schedule
  - Everyday looks exactly the same
  - Schedule repeat itself every 24 hours
Space-time network

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A network for each fleet type

- Node: arrival station (of a flight) at ready time, departure station at departure time
- Arc:
  - from departure to arrival of each flight
  - each node to adjacent node on time line at same station (ground arc)
Balanced space-time network
Unbalanced network
Basic fleet assignment problem

Given

- Flight schedule
- Number of aircraft by equipment type
- Turn time by fleet type at each station
- Operating costs and potential revenue of flights, by fleet type

Find

Maximize profit when assigning aircraft types to scheduled flights, such that:

- Every flight is covered by exactly one fleet type
- Conservation of flow of aircraft is achieved
- The number of aircraft used does not exceed the number available (in each aircraft type)
### Basic fleet assignment problem

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Fleet assignment model

Formulation in English

- **Max**: Leg-based contribution
Formulation in English

- **Max:** Leg-based contribution
- **Subject to:**
  - Each flight is covered by an aircraft type
  - Aircraft flow in the network is conserved
  - Number of aircraft used does not exceed the number available
### Inbound flights at Atlanta

<table>
<thead>
<tr>
<th>Time</th>
<th>Flight Time</th>
<th>City</th>
<th>Flight #</th>
<th>Aircraft type</th>
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</thead>
<tbody>
<tr>
<td>6:00am</td>
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<td>Boston</td>
<td>709</td>
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Example: single flight
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Max \[ C_{B747,1} \cdot f_{B747,1} + C_{B777,1} \cdot f_{B777,1} + C_{B767,1} \cdot f_{B767,1} + C_{B757,1} \cdot f_{B757,1} \]
Constraints 1: coverage

\[
\text{max} \quad C_{B747,1} \cdot f_{B747,1} + C_{B777,1} \cdot f_{B777,1} + \\
C_{B767,1} \cdot f_{B767,1} + C_{B757,1} \cdot f_{B757,1} \\
\text{s.t.} \quad f_{B747,1} + f_{B777,1} + f_{B767,1} + f_{B757,1} = 1 \\
f_{B747,1}, f_{B777,1}, f_{B767,1}, f_{B757,1} \in \{0, 1\}
\]
Airline network is a “network”, it needs conservation of flow
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\[ y_{B777, SYD, 1600^-} + f_{B777, 2} - y_{B777, SYD, 1600^+} = 0 \]
Airline network is a “network”, it needs conservation of flow

\[ y(B777, SYD, 1600^-) + f_{B777,2} - y(B777, SYD, 1600^+) = 0 \]

At any event node \((k, o, t)\)

\[ y(k, o, t^-) + \sum_{i \in \text{In}(k, o, t)} f_{k,i} - y(k, o, t^+) - \sum_{i \in \text{Out}(k, o, t)} f_{k,i} = 0 \]
Question: “How do we count aircraft in the network (using given variables)?”
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Flow conservation
Constraints 3: count

**Question:** “How do we count aircraft in the network (using given variables)?”

**Flow conservation**

**Practice**

Take a snapshot a few hours after midnight because most aircraft is on the ground

\[
\sum_{o \in O} y(k,o,t_n) + \sum_{i \in CL(k)} f_{k,i} \leq N_k, \forall k \in K
\]
FAM formulation

\[
\begin{align*}
\text{max} & \quad \sum_{i \in L} \sum_{k \in K} C_{k,i} \cdot f_{k,i} \\
\text{s.t.} & \quad \sum_{k \in K} f_{k,i} = 1 \\
& \quad y_{k,o,t}^- + \sum_{i \in \text{ln}(k,o,t)} f_{k,i} - y_{k,o,t}^+ - \sum_{i \in \text{Out}(k,o,t)} f_{k,i} = 0, \forall k, o, t \\
& \quad \sum_{o \in O} y_{k,o,t_n} + \sum_{i \in \text{CL}(k)} f_{k,i} = N_k, \forall k \in K \\
& \quad f_{k,i} \in \{0, 1\}, y_{k,o,t} \in \mathbb{Z}^+ \end{align*}
\]
A major US airline
- 2000 daily flights, 12 fleets, 400 aircraft, 300 cities
lead to constraint matrix with
- \(~ 72000\) columns (variables)
- \(~ 50000\) rows (constraints)
## Pros & Cons

<table>
<thead>
<tr>
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<th>Cons</th>
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<tbody>
<tr>
<td>- Compact formulation</td>
<td>- Assume deterministic demand</td>
</tr>
<tr>
<td>- Superb solvability</td>
<td>- Ignore network effect</td>
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<td>- Reasonable approximation of reality</td>
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Other approaches

- ILP cannot handle complex constraints
- Use other (meta-)heuristics to find good solution (from initial FAM solution)